Bundle-Size Pricing as an Approximation to Mixed Bundling

Based on Chu, Leslie, and Sorensen (2011)

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Overview

Outline

Main issue

- Pricing problem of a multiproduct firm facing consumers who may purchase more than one
- Examples: professional sports teams, online music stores

Pricing strategies

- Different pricing strategies for bundling

Initials	Name	Prices	Description
UP	Uniform pricing	1	Each product sold separately at a uniform price
PB	Pure bundling	1	Only option for consumers is the full bundle
CP	Component pricing	K	Each product sold separately at a different price
BSP	Bundle-size pricing	K	Prices depend only on number of purchased products
MB	Mixed bundling	$2^K - 1$	Separate prices for every possible combination

Pricing strategies

- Different pricing strategies for a movie theater

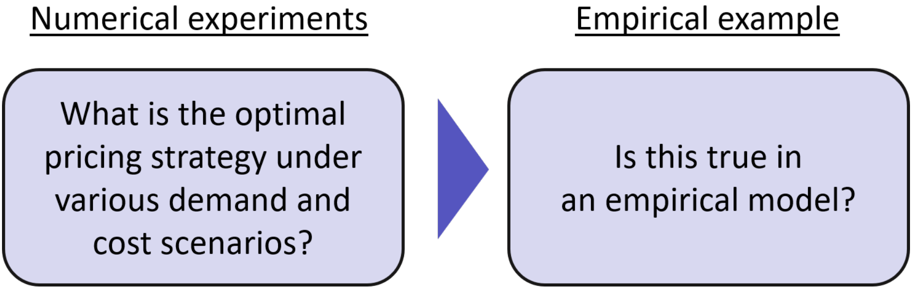
Uniform pricing	Toy Story $7	Star Wars $7	
Pure bundling	Toy Story Star Wars $15		
Component pricing	Toy Story $10	Star Wars $6	
Bundle-size pricing	1 movie $7	2 movies $12	
Mixed bundling	Toy Story $10	Star Wars $6	Toy Story Star Wars $15

Prior literature

Prior Literature		This paper
Mixed bundling is always more profitable than CP. (Adams and Yellen, 1976; McAfee et al., 1989)	→	MB becomes impractical as the number of goods increases.
In some cases, PB may also be more profitable than CP. (Bakos and Brynjolfsson, 1999; Armstrong, 1999; Fang and Norman, 2006)	→	Even in these cases, there can be other pricing schemes that will do even better.

Question

- Question: Do pricing schemes exist that involve few enough prices to be feasible, and that tends to yield profits close to the mixed bundling level?
- Answer: Bundle-size pricing can approximate mixed bundling.



Outline

Assumptions

- A monopolist firm producing two goods ($MC = 0$)
- Consumers purchase one or zero units of each product.
- Consumers' valuations for a bundle equal the sum of their valuations for the bundles' component products ($v = v_1 + v_2$)
- No resale.
- Consumers' valuations for good 1 & 2

$$v_1 \sim U[0, \theta], v_2 \sim U[0, 1]$$

$$v_1, v_2 \text{ uncorrelated}$$

Example

Consumers' valuations for good 1&2

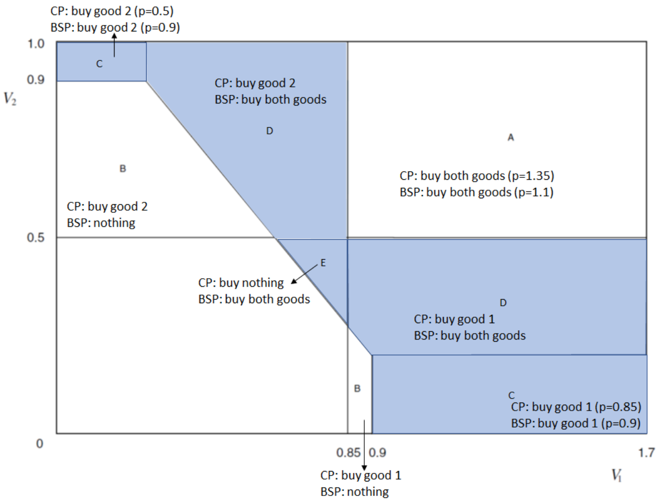
$$v_1 \sim U[0, \theta], v_2 \sim U[0, 1]$$

If $\theta = 1.7$, optimal prices for each pricing schemes are:

	P_1	P_2	P_B
CP	0.85	0.5	-
BSP	0.9	0.9	1.1
MB	1.13	0.67	1.18

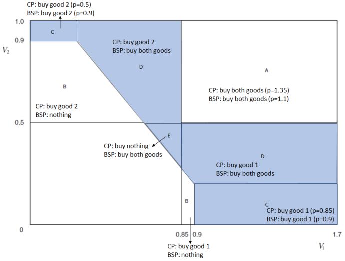
- Price for a single-good bundle under BSP (0.9) lies between two single-good prices under MB.
- BSP prices closely approximate MB prices for large-sized bundles.
- BSP is 5.6% more profitable than CP.

Separation of Consumers under CP and BSP



BSP extracts more surplus from consumers than CP in shaded areas.

Results



Four points to take away

- BSP is more focused than CP on getting consumers to purchase multiple goods.
- Negative correlation in consumers' valuations may increase the relative profitability of BSP.
- Diminishing marginal utility can reduce the profitability of BSP and CP but it is possible that CP is more affected than BSP.
- Complexity of BSP pricing problem

Outline

Setting

Assumptions

- Consumer i's utility from purchasing bundle j

$$V_i' D_j - p_j$$

  where V_i is a $K \times 1$ vector of valuations for the firm's K products, D_j is a $K \times 1$ vector of binary indicators for which of the K products are included in bundle j, and p_j is the price of bundle j
- Heterogeneous consumers: $V_i \sim F$
- Free disposal

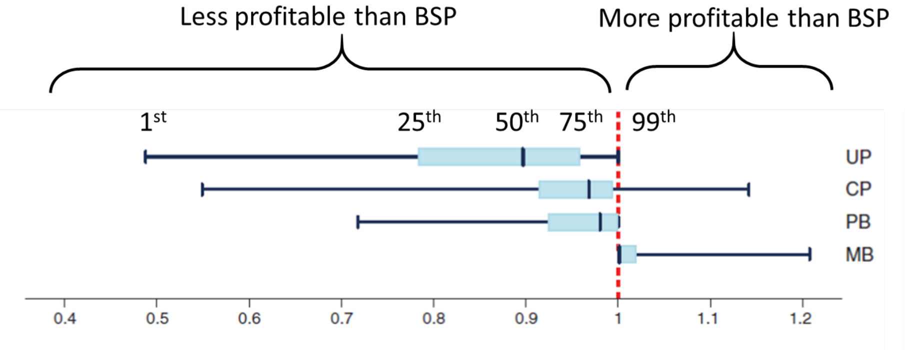
Numerical analysis with continuous types and more than two goods

- They examine computational experiments and solve for optimal pricing strategies.
- They allow variations in
  1. Number of goods: $K = 2, 3, 4, 5$
  2. Cost: $MC = 0, MC > 0$ & equal, $MC > 0$ & differing, $MC = 0$ & binding capacity constraint
  3. Taste distribution: exponential, logit, lognormal, normal, uniform
  4. Parameter values: 220 parameter combinations for each class of distributions

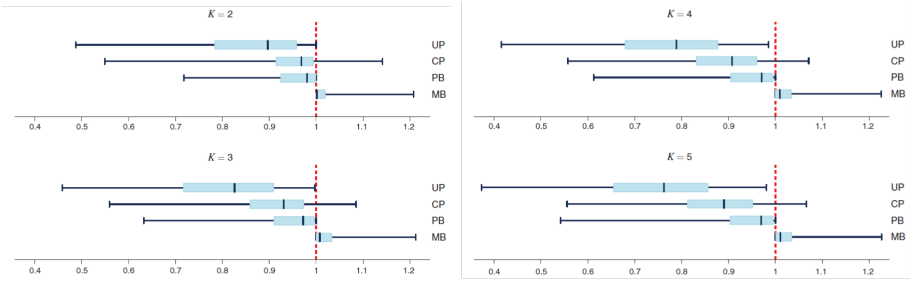
     $\Rightarrow$ Limitation: Results depend on the specific parameters they choose.

Digression: How to read box plots

Distributions of profits for each pricing strategy relative to BSP



Number of goods



1. BSP tends to be more profitable than CP (90% out of 71,360 experiments).
2. BSP tends to obtain profits that are within 1% of profits from MB (on average, 98% of the MB profits).
3. Increasing the number of goods tends to favor bundling over CP. $\Rightarrow$ Heterogeneity-reducing effect

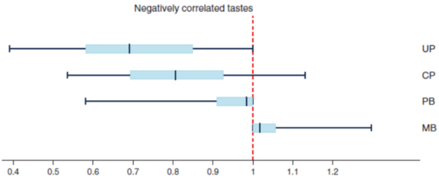
Heterogeneity-reducing effect

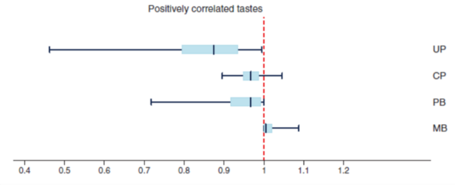
- Number of goods in a bundle increases.
  $\Rightarrow$ Variance in consumers' valuations for the bundle decreases.
  $\Rightarrow$ Under MB, the optimal prices for different bundles of a given size K are not very different.
  $\Rightarrow$ BSP is a good approximation when K is large.
- Price differences by bundle size

Bundle size	Average price differences $\lvert p_{CP} - p_{MB}\rvert / p_{MB}$	Average price differences $\lvert p_{BSP} - p_{MB}\rvert / p_{MB}$
Panel B. K = 4		
1	0.410	0.674
2	0.242	0.242
3	0.167	0.095
4	0.180	0.036
Panel C. K = 5		
1	0.457	0.802
2	0.300	0.328
3	0.212	0.154
4	0.171	0.079
5	0.190	0.038

- Small-sized bundles: CP is a better approximation for MB.
- Large-sized bundles: BSP is a better approximation for MB.

Correlation



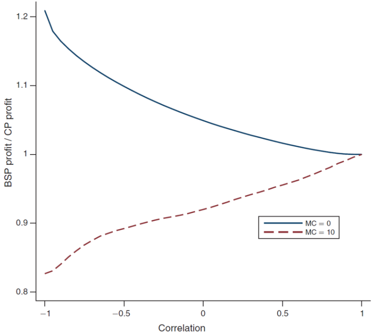


- Usually, the more negatively correlated are consumers' tastes, the more profitable is BSP relative to CP.
- Negatively correlated tastes: CP can be up to 12% more profitable than BSP, and up to 47% less profitable.
- Positively correlated tastes: Range of possibilities is much smaller at *both extremes*.

  $\Rightarrow$ This means that in some cases, negative correlation reduces the profitability of BSP relative to CP. Why?

Correlation

BSP profits relative to CP profits, as a function of correlation in consumers' tastes across products

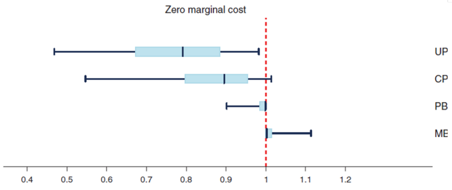


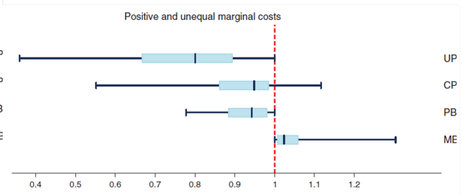
- When MC=0, the more negatively correlated are consumers' tastes, the more profitable is BSP relative to CP.
- When MC=10, the more negatively correlated are consumers' tastes, the less profitable is BSP relative to CP.

Correlation

- Question: Why does the negative correlation favor BSP?
- Answer: Heterogeneity-reducing effect: High valuations for some products in the bundles are offset by low valuations for other products in the bundle.
- Question: Then why does negative correlation reduce the profitability of BSP relative to CP when MC is high?
- Answer: If $MC > V_i$, then negative correlation makes it less likely that a consumer values more than one of the firm's products above cost. $\Rightarrow$ Thus, inducing a consumer to purchase an additional product may actually lower the consumer's valuation of the bundle relative to its cost.

Costs and capacity constraints





- Interaction effect between MC and correlation: Increases in marginal costs are most harmful to BSP profitability when tastes are negatively correlated.
- Even when MCs are higher, BSP is more profitable than CP in 83% of experiments.
- BSP is less sensitive than PB to increases in MCs.

Demand Asymmetry

- Intuitively, increasing demand asymmetry across products may favor CP over BSP, because the ability to set different prices for different goods become increasingly important. Is this true?
- A simple example with 2 goods, zero MC, 2 consumers

	v_1	v_2
A	2	0
B	0	1

	P_1	P_2	π
CP	2	1	3
BSP	1 (or 2)	1 (or 2)	2

	v_1	v_2
A	20	0
B	0	1

	P_1	P_2	π
CP	20	1	21
BSP	20	20	20

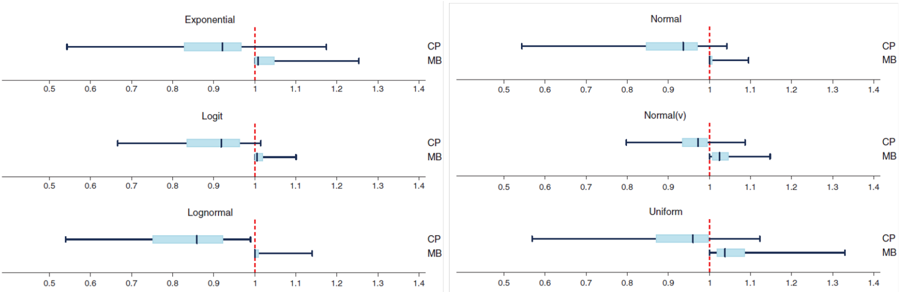
- As demand becomes more asymmetric, CP remains more profitable than BSP, but the absolute advantage does not change (=1), and in percentage terms BSP profits actually get closer to CP profits.

Parametric families

Alternative taste distributions

Name	Description
Exponential	Marginal distributions of the nu_{ik} are exponential, with means between 0.2 and 2.0
Logit	Marginal distributions of the ν_{ik}s are extreme value, with means between 0 and 2.5 and scale parameter=0.25
Lognormal	ν_k is a multivariate lognormal random variable; $\log(v_{ik})$s have variance 0.25 and means between -1.5 and 1
Normal	ν_i is a multivariate normal random vector; ν_{ik}s have variance 0.25 and means between -1 and 2.5
Normal(ν)	ν_i is a multivariate normal random vector; ν_{ik}s have mean zero and variances between 0.25 and 1.75
Uniform	Marginal distributions of the ν_{ik}s are uniform on $[0, a_k]$, with a_k between 0.4 and 4

Parametric families



- Logit/Lognormal: BSP is almost always more profitable than CP.
- Exponential: When MC=0, BSP is always more profitable than CP.
- Normal: Either CP or BSP may be the most profitable under any assumption on costs.
  ⇒ They assume normally distributed tastes in next section.

Summary of combined effects

Regression analysis of (BSP profit/CP profit)

	Exponential	Logit	Lognormal	Normal	Normal(v)	Uniform	All combined
Number of products dummies:							
$K = 3$	0.069	0.031	0.068	0.042	0.036	0.034	0.038
	(0.003)	(0.002)	(0.002)	(0.004)	(0.001)	(0.003)	(0.002)
$K = 4$	0.122	0.053	0.120	0.078	0.070	0.064	0.071
	(0.003)	(0.002)	(0.002)	(0.004)	(0.001)	(0.003)	(0.002)
$K = 5$	0.160	0.071	0.159	0.106	0.096	0.092	0.098
	(0.003)	(0.002)	(0.002)	(0.004)	(0.001)	(0.003)	(0.002)
MC scenario dummies:							
Positive, equal	−0.064	−0.009	−0.023	−0.012	−0.022	−0.039	−0.028
	(0.005)	(0.003)	(0.004)	(0.007)	(0.002)	(0.005)	(0.003)
Positive, unequal	−0.159	−0.002	−0.096	−0.010	−0.033	−0.119	−0.070
	(0.005)	(0.003)	(0.004)	(0.007)	(0.002)	(0.005)	(0.003)
Capacity constraints	−0.179	−0.027	−0.029	−0.016	−0.045	−0.110	−0.068
	(0.005)	(0.003)	(0.004)	(0.007)	(0.002)	(0.005)	(0.003)
Covariance structure dummies:							
Negative	0.504	0.237	0.550	0.370	0.063	0.499	0.325
	(0.006)	(0.003)	(0.004)	(0.008)	(0.002)	(0.006)	(0.003)
Positive	−0.148	−0.073	−0.140	−0.075	−0.053	−0.093	−0.097
	(0.005)	(0.003)	(0.004)	(0.007)	(0.002)	(0.005)	(0.003)
Cov. × MC interactions:							
Neg. × (pos/eq.)	−0.131	−0.007	0.006	−0.013	−0.046	−0.042	−0.039
	(0.006)	(0.004)	(0.005)	(0.010)	(0.002)	(0.006)	(0.004)
Neg. × (pos/uneq.)	−0.377	0.058	−0.059	0.034	−0.044	−0.142	−0.088
	(0.006)	(0.004)	(0.005)	(0.010)	(0.002)	(0.006)	(0.004)
Neg. × (cap. constr.)	−0.374	−0.099	−0.224	−0.157	−0.065	−0.286	−0.201
	(0.006)	(0.004)	(0.005)	(0.010)	(0.002)	(0.006)	(0.004)
Pos. × (pos/eq.)	0.047	0.007	0.007	0.008	0.017	0.028	0.019
	(0.006)	(0.004)	(0.005)	(0.010)	(0.002)	(0.006)	(0.004)
Pos. × (pos/uneq.)	0.104	−0.000	0.053	0.007	0.018	0.072	0.042
	(0.006)	(0.004)	(0.005)	(0.010)	(0.002)	(0.006)	(0.004)
Pos. × (cap. constr.)	0.165	0.023	0.032	0.022	0.030	0.086	0.060
	(0.006)	(0.004)	(0.005)	(0.010)	(0.002)	(0.006)	(0.004)
Asymmetry in product valuations:							
Asymmetry	−0.063	0.007	−0.082	−0.006	−0.082	−0.107	−0.033
	(0.011)	(0.005)	(0.006)	(0.008)	(0.004)	(0.005)	(0.003)
Asymmetry^2	−0.025	−0.022	0.006	−0.015	−0.017	0.032	−0.003
	(0.009)	(0.003)	(0.004)	(0.003)	(0.004)	(0.002)	(0.002)
Asymmetry × neg. corr.	−0.239	−0.068	−0.311	−0.133	0.013	−0.184	−0.094
	(0.007)	(0.003)	(0.004)	(0.005)	(0.003)	(0.003)	(0.002)
Constant	1.171	1.077	1.189	1.074	1.053	1.139	1.115
	(0.004)	(0.003)	(0.003)	(0.006)	(0.001)	(0.004)	(0.002)
N	10,704	10,704	10,704	10,704	10,704	10,704	64,224
R^2	0.790	0.790	0.875	0.492	0.701	0.744	0.477

- Question: How much of the variation in the relative profits of BSP and CP can be explained by a combination of all these factors?
- Answer: Run regressions using our experimental outcomes.

Summary of combined effects

Regression analysis of (BSP profit/CP profit)

	Coefficients
Number of products	
$K = 3$	0.038
$K = 4$	0.071
$K = 5$	0.098
MC scenario	
Positive, equal	-0.028
Covariance structure	
Negative	0.325
Cov.$\times$ MC	
Neg.$\times$ (pos/eq.)	-0.039
Post.$\times$ (pos/eq.)	0.019
N	64,224
R^2	0.477

- Dependent variable: ratio of BSP profit to CP profit
- Profitability of BSP relative to CP increases with K and decreases with MCs.
- Negative correlation favors BSP, but significantly less when MCs are high (Neg. $\times$ MC interactions).

Outline

Appealing features of play ticket data

- Plays differ in their overall popularity, making it plausible that CP would be a sensible pricing strategy.
- Many consumers attend more than one play, making it plausible that bundling strategies may also be profitable.
- Individuals do not consume multiple units of the same play.
- Assumption of no demand or cost interdependencies is reasonable.
- There is no significant resale activity.

Data summary

- All ticket sales for *TheatreWorks*' 2003-2004 season
- 229 performances of eight different plays, 69,207 tickets sold

Play	Type	Number of performances	Average attendance	Ticket sales (subscription)	Ticket sales (nonsubscription)
A Little Night Music	Musical	30	294.87	7,018	1,828
All My Sons	Drama	33	233.85	6,826	891
Bat Boy	Musical	30	263.93	6,782	1,136
Memphis	Musical	30	352.40	6,999	3,573
My Antonia	Drama	26	312.38	7,002	1,120
Nickel and Dimed	Drama	26	343.62	6,800	2,134
Proof	Drama	25	319.88	6,885	1,112
The Fourth Wall	Comedy	29	313.83	7,385	1,716
Total		229	302.21	55,697	13,510

Purchase options

- Nonsubscription: individual play ticket at a uniform price
- Subscription
  - The full eight-play season
  - Any combination of five plays
  - A prespecified bundle of three plays

Purchase option	Price per play ($)	Number of consumers
Nonsubscription:		
1 play	40.80	8,131
2 plays	40.80	1,409
3 plays	40.80	555
4 plays	40.80	224
Subscription:		
3-play bundle	36.20	205
5-play pick	37.00	2,794
8-play bundle	34.55	5,139

Outline

Setting

- The firm offers $j = 1, \ldots, J-1$ bundles containing combinations of the $k = 1, \ldots, K$ products.
- Consumers maximize utility.

$$u_{ij} = \begin{cases} V_i' D_j - \alpha p_j : & j = \{1, \ldots, J-1\} \\ 0 \quad : & j = J \end{cases}$$

where V_i is a $K \times 1$ vector of valuations for the firm's K products, D_j is a $K \times 1$ vector of binary indicators for which of the K products are included in bundle j, p_j is the price of bundle j, and $\alpha > 0$ measures the sensitivity to price.

Setting

- Two classes of consumers: theater lovers and regular consumers

$$V_i = \max\{\theta_i + \epsilon_i, 0\}, where$$
$$\theta_i = \begin{cases} \bar{\theta} : & \text{probability } \lambda \\ 0 : & \text{probability } (1-\lambda) \end{cases}$$
$$\epsilon_i \sim N(\mu, \Sigma)$$

  where μ is a $K \times 1$ vector of means, Σ is a $K \times K$ variance-covariance matrix, and $\bar{\theta}$ is a scalar additive component.

- Modeling the demand for 220 alternatives (219 different bundles + 1 outside option)

Estimating method

- We estimate 39 parameters: $\alpha, \Sigma, \bar{\theta}, \lambda$, M (market size)
- For a given set of parameters, Θ,
  1. Draw n_s simulated consumers from distribution of product valuations.
  2. Compute the optimal bundle choice for each consumers.
  3. Compute the optimal prices.
- Choose the estimators such that

$$\text{Predicted market shares} \approx \text{Actual market shares}$$

conditional on predicted prices $\approx$ actual prices

Outline

Estimated coefficients

TABLE 8A—ESTIMATED COEFFICIENTS

	Covariances (Σ) (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	1.0000							
(2)	0.9357	1.2200						
(3)	1.2125	1.4150	1.7208					
(4)	0.9793	1.3381	1.4859	3.2685				
(5)	0.8743	1.1055	1.2207	1.7920	1.4308			
(6)	1.1602	1.3451	1.6211	1.9090	1.3801	1.9610		
(7)	0.7886	1.0600	1.2199	1.7924	1.2127	1.4517	1.5086	
(8)	1.1133	1.2873	1.5878	2.5509	1.5597	1.9529	2.0171	3.0732

	Estimate	SE
Price sensitivity (α)	4.5937	(0.0851)
Probability of theater lover (λ)	0.0805	(0.0060)
Increment for theater lovers ($\bar{\theta}$)	2.0561	(0.1665)
Market size	36,055	(972)

Notes: Standard errors for Σ are in Table 8B. All parameter estimates are significant at the 1 percent level.

- Covariance matrix: correlation structure conditional on being a theater lover, or conditional on not being a theater lover
- Prob. of being a theater lover = 0.081
- Theater lovers' utility for any single play is higher than for regular consumers by 2.06 times the SD of the conditional valuation of play 1.

Estimated coefficients

Predicted market shares for bundles

	Non-theater lovers	Theater lovers
Outside option	63%	9.0%
pick-5 bundle	8.9%	14%
all-8 bundle	9.9%	66%

- The large magnitude of the increment to utility for theater lovers ($\bar{\theta} = 2.06$) suggests that large-sized bundles are disproportionately purchased by theater lovers.
- The nonmonotonicity of predicted market shares with respect to bundle size is consistent with the high degree of correlation in the estimated distribution of tastes for individual plays: consumers tend to like either most of the plays, or none at all.
- Overall, the model fits the data fairly well.

Outline

Alternative pricing strategies

- Using the estimated demand model, compute profits and consumer surplus under each of UP, PB, CP, BSP, and MB.
- BSP attains 0.9% higher profit than CP
- BSP attains 98.5% of the profit from MB.
  $\Rightarrow$ MB requires 255 prices while BSP requires 8 prices.
  $\Rightarrow$ BSP encourages consumers to purchase multiple plays.

	UP	PB	TW	CP	BSP	MB
p_1	35.60		44.55	27.79	56.41	48.25
p_2				30.07	46.92	43.08
p_3			38.01	34.67	41.12	40.57
p_4				44.08	37.72	38.68
p_5			36.68	31.46	36.80	38.11
p_6				38.89	35.04	36.54
p_7				33.23	34.01	35.23
p_8		30.81	33.30	37.90	32.89	34.29
Revenue	66.85	63.67	67.57	67.81	68.42	69.50
CS	55.03	54.37	54.02	55.88	54.75	52.62

Outline

Conclusion

Numerical experiments		Empirical example
BSP tends to attain nearly the same level of profits as MB in a broad range of demand and cost scenarios.	▶	BSP is 0.9% more profitable than CP.
BSP tends to be more profitable than CP (even with demand asymmetry and with positively correlated tastes).	▶	BSP attains 98.5% of MB profits.

- Central theme: Bundle-size pricing tends to attain nearly the same profits as mixed bundling and is almost certainly more profitable than either component pricing or pure bundling.

Conclusion

- Question: Do pricing schemes exist that involve few enough prices to be feasible, and that tends to yield profits close to the mixed bundling level?
- Answer: Bundle-size pricing can approximate mixed bundling.

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Q & A

Thank you.